Problem Set 3
due **Tuesday, Feb. 11 in class**
(or in my mailbox beforehand)

Decisions, Games & Rational Choice

Exercise 1 (15 pts.) [adapted from R. Jeffrey.] You are suffering from a disease that, according to your manifest symptoms, is either disease A or disease B. A happens to be 19 times as common as B, and this demographic statistic is the only evidence that is relevant (without further investigation) to determining which of the two you have. The two diseases are both fatal if untreated, and there is a treatment available for both. Unfortunately, it is dangerous—in fact fatal—to combine the two treatments. There is, however, a test which your doctor has ordered, which yields a definitive diagnosis, and the diagnosis is correct, both with cases of A and cases of B, 90% of the time. (That is, when the patient has A, the test result is right in 90% of the cases, and when the patient has B, the test result is right in 90% of the cases.) The test reports that you are suffering from disease B. Which treatment should you take, and why?

Exercise 2 (15 pts.) Jones is deciding whether to go to college or not. All Jones cares about is getting money (and each dollar is worth as much to him as the next). If he goes to college, he applies to jobs when he's done, otherwise he applies immediately. In both cases he'll start by applying to Job A, which pays $60,000 a year. But it's allotted selectively. About 50% of college graduates who applied got the job. Only 5% of non-graduates who applied got it. If Jones doesn't get Job A after applying, he'll work Job B, which has no qualifications and limitless positions, and pays $30,000 a year. College is free, but it takes time. Jones has ten years of solid work in him. But if he goes to college, that reduces it to seven. What should Jones do if he were to maximize expected utility? (Set up the calculation, do it, and show your work).

Exercise 3 (10 pts.) Show that if d is probabilistically independent of e, then e is probabilistically independent of d.

Exercise 4 (10 pts.) Three men—A, B, and C—were put in jail for speaking out against the king. The king announced that as punishment he had selected two of the three randomly to be hanged, the other set free. The three men were placed in separate cells, unable to communicate, until the time of the executions. A, horrified at a mere 1/3 chance of survival, spoke to his guard, who knew which men were destined for death: "Look, one man other than me is going to be executed for sure. So you won't really be telling me anything about my own situation if you give me the name of that man who is getting the noose." The guard, after some reflection, thought this was right, and so told him "C is a goner." A sighed in relief. After all, now the odds had changed: either B would survive or he would, and they both had an equal shot. So he could rest assured that he had a 50% of living.

Is A's reasoning that he has a 50% chance of survival correct? If so, explain why. If not, explain how likely A should think it is (after his talk with the guard) that he will be executed, and also how likely he should think it is that B will be executed.