PROBLEM SET 2 due **Tuesday, February 4** In class (or in my mailbox before)

RATIONAL CHOICE

Exercise 1 (15 pts.) You're playing a lottery which doles out many tickets, each of which has a 1/100 chance of winning a prize. Whether one ticket wins or not is completely independent of whether another ticket wins a prize (so it's possible to win multiple prizes). What you want most is to win *at least one prize*.

(i) Suppose you have 1 ticket only. If you buy a second ticket, how much will it *improve* your chances of winning at least one prize. That is, what's the difference between the chances that you'll win at least one prize if you have two tickets and the chance that you'll win at least one prize if you have just one?

Hint: You'll be needing the rule for disjunction. And you'll need to figure out the probability that several tickets could win at once. Since whether one ticket wins is independent of another:

 $P(\text{ticket 1 wins \& ticket 2 wins}) = P(\text{ticket 1 wins}) \times P(\text{ticket 2 wins})$

(ii) Suppose you have 100 tickets. If you buy another, will it improve your chances of winning at least one prize *more* than your answer to (i), the *same* as in (i) or *less* than in (i)? If it is more, is it much more or not? If less, much less or not? Don't do the actual calculation here—just think about it. Then explain your answer in intuitive terms. **Exercise 2** (10 pts.) Suppose Sally has credences which violate the first axiom of probability and is willing to place bets according to her credences as discussed in class. Show how to construct a Dutch Book for Sally. (Don't worry about what it means to assign 'negative probability' to an event. Just assume that probabilities are correlated with betting behavior according to the assumption involved in the Dutch Book Theorem. And assume that a bet with a 'negative payoff', like a "gain of -\$n", is just a loss of \$n.)

Exercise 3 (15 pts.) Suppose you draw a card at random from a standard deck of 52 cards. Use the definition of conditional probability to show ascertain the following. (Show your work). ["Ace" = "the card you draw is an Ace", "Heart"="the card you draw is a heart", "Red" = "the card you draw is red"]

- (i) P(Heart | Ace)
- (ii) P(Ace | Heart)
- (iii) P(Ace & Heart | Red)

Exercise 4 (10 pts.) Prove that if one's prior credences P satisfy all the axioms of probability, then (assuming the rule of Conditionalization) P_E satisfies the second axiom. That is, on the relevant assumptions, show

$$P_E(s) = 1$$
 if s is 'certain'

Be precise, and make clear where you employ the assumption that P satisfies the axioms (and say which of those axioms mattered!).