Handout 7
Puzzles about Conditionalization \& Maximizing Utility I

## $\mathbf{R}^{\text {Decisions, Games \& }}$ ational choice

## Ellsberg Paradox

There are two strategies for prescribing rational choice under uncertainty (as opposed to under risk).
(A) Transform the problem from one involving uncertainty into one involving risk. Since probabilities are treated subjectively, this just means taking a stab estimating the probabilities. Then maximze utility as normal.
(B) Supplement your theory with special rules for cases of uncertainty (e.g., maximax, maximin, etc.)

The following puzzle is a useful one for thinking about rational choice under uncertainty.

Ellsberg. An urn contains 30 red balls and 60 balls that are either black or white (but you are not told the proportion). A ball will be picked at random from the urn and you are offered the following choices:

A: receive $\$ 100$ if a red ball is drawn
B: receive $\$ 100$ if a black ball is drawn

What should you choose? What if you are offered the following:
C: receive $\$ 100$ if either a red or white ball is drawn.
D: receive $\$ 100$ if either a black or white ball is drawn.

## Allais' Paradox

Allais. You're offered a choice between two lotteries, A or B.

A: $\quad 11 \% \$ 1,000,000$
89\% \$0
B: $\quad 10 \% \$ 5,000,000$
90\% \$0

Suppose instead you're offered these options.

C: $\quad 100 \% \$ 1,000,000$
D: $\quad 89 \% \quad \$ 1,000,000$
$10 \% \quad \$ 5,000,000$
$1 \% \quad \$ 0$

## The St. Petersburg Paradox

St. Petersburg. I'll toss a coin over and over. Each time it comes up heads, your payoff doubles. Once the coin lands tails the game is over. So the payoffs are
\$1 if no heads come up before the first tails
$\$ 2$ if exactly one heads comes up before the first tails
$\$ 4$ if exactly two heads comes up before the first tails
$\$ 8$ if exactly three heads comes up before the first tails
$\$ 16$ if exactly four heads comes up before the first tails
...etc...
How much would you pay me to play this game?

