Best Responses & Nash Equilibrium

Let's introduce some more definitions.

> A ***strategy*** is a "complete" series of moves made by a single player in a game.

For now we're looking at games with one move per player, so a strategy just is a move. In other games where there are many opportunities to move (e.g. tic-tac-toe), a strategy essentially a plan for how to move initially, and in response to every possible opponent's move.

> A player's strategy is a ***best response*** to other players' strategies if it is among the strategies that yields the best outcome holding the other players' strategies fixed.

Consider, for example

Column

Row

	C1	C2
R1	6 1	3 5
R2	3 2	4 0

What's Row's best response to C1? R2. This gives a utility of 2 over 1.
What's Row's best response to C2? R1. This gives a utility of 5 over 0.

It seems: *if* you're very confident your opponent will choose strategy S, *then* you should choose a best response to S. Now,

> A ***strategy profile*** is set of strategies for every player in a game.

We'll usually put strategies in angled brackets. So <R1, C2> is a possible strategy profile for the game above. Finally,

> A strategy profile is a ***Nash equilibrium*** if every strategy in the profile is a best response to the other strategies in the profile.

In a Nash equilibrium, no player can get any more utility by changing their strategy *alone.* First let's practice finding "pure strategy" Nash equilibria (these involve strategies in which no one randomizes their choices). Then we'll say why they matter.

The simplest way to find some equilibria in a simple game is by listing best responses and finding which strategies are best-responses to each other.

Column

Row

	C1	C2	C3
R1	2 0	3 2	3 4
R2	1 11	2 3	0 0
R3	3 0	0 1	0 8

Nash Equilibrium & Dominance

Let's apply the concept of Nash-Equilibrium to our old favorite, the Prisoner's Dilemma.

Column

Row

	C1	C2
R1	1 1	2 -1
R2	-1 2	0 0

You'll see the Nash-equilibrium is the result of applying dominance reasoning. This isn't a coincidence. *No strategy in a Nash equilibrium is ever strongly dominated*. (Why?)

But a corresponding result doesn't hold for weak dominance. Consider:

Column

Row

	C1	C2
R1	1 1	0 0
R2	0 0	0 0

Ok, so why *care* about Nash Equilibria? Consider games where it's common knowledge that choices of strategy don't causally influence each other. And suppose our usual:

> (C) All players are rational, and this is common knowledge.

Does (C) entail that Nash equilibrium strategies will be played? It would if (C) entailed (C').

> (C') Each player can reliably predict every other player's strategies, and this is common knowledge.

If (C') were true, then we'd have a great argument that Nash equilibria will always be played by reductio:

> Suppose <A, B> is played and is not a Nash equilibrium. Then player one (say) might benefit from switching their strategy to <A',B>. But by (C'), player one was in a position to know B was being played, and was rational, and hence playing to optimize their outcomes, but she didn't optimize her outcomes.

One might think (C) *entails* (C'). Why? Well you might think that maximally rational people would do the exact same thing, when placed in the same circumstances. If I'm rational, and I know you're rational, maybe I can figure out what it is that you're going to do. All I have to do is ask myself: What would I do in your shoes? So maybe (C') is true on its own.

Another reason to be interested in Nash equilibria is that in some circumstances strategies "gravitate" towards equilibria in repeated games. We already saw a potential example of this with our previous "2/3rds game".